# Algebra (basic rules) 

E=evidence<br>$\mathrm{Hd}=$ defence hypothesis<br>Hp=prosecution hypothesis<br>S=suspect<br>V=victim

## The numerator (forget peak area for the time being)



- Consider a simple 2 person mixture - 1 locus only consisting of 2 heterozygotes
- The suspect is $a b$
- There must be another (unknown person) who is cd


## The numerator (forget peak area for the time being)


-The prosecution say: the DNA has come from the suspect and one unknown person
-The defence say: The DNA has come from 2 unknown people - How do we evaluate this evidence?

## The numerator (forget peak area for the time being)


-The prosecution say: the DNA has come from the suspect and one unknown person
-IF this is true then we would expect $a b$ to be present with probability
$\cdot \operatorname{Pr}=1$. The chance of seeing an unknown person of type cd is $2 p_{c} p_{d}$
$\cdot \operatorname{Pr}(E \mid S, U)=2 p c p d$
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## The denominator

- The defence say: this could | Individual | Individual | products |
| :--- | :--- | :--- | have come from any two random individuals
- Work out all pairwise combinations from abcd and their probabilities

These are the reverse combations

| ad | bc | 4abcd |
| :---: | :---: | :---: |
| cd | ab | 4abcd |
| bd | ac | 4 abcd |
| bc | ad | 4abcd |
|  | Sum of <br> products | 24 abcd |

## Now we form the LR

## $L R=\frac{\operatorname{Pr}(E \mid S, U)}{\operatorname{Pr}\left(E \mid U_{1}, U_{2}\right)}$ $=\frac{2 \varepsilon d}{12^{24 a b c d}}$ <br> $$
=\frac{1}{12 a b}
$$

## All Calculations follow the same basic rules as described

- Different hypotheses?
- More on conditioning
- The top bit of the equation belongs to the prosecution
- The bottom bit of the equation belongs to the defence.
- Lets change the hypotheses


## The numerator (forget peak area for the time being)



- The profile is from a semen contaminated vaginal swab
-The suspect is ab and the victim is cd
-Because it is reasonable to suppose that alleles from the victim are on the swab we can condition on this


## Conditioning on a victim

- The prosecution hypothesis (S+V) is completely explained hence the probability of the evidence is $\mathrm{Pr}=1$.
- The defence hypothesis is U (still) +V.
- The LR is therefore:

$$
\frac{1}{2 a b}
$$

- This is the same as for a non-mixed sample comprising the suspect alone.
- An illustration of simplification by ‘subtraction’
- Subtraction is an important part of reporting mixtures but care must always be taken to ensure that we do not remove alleles simply because they are 'inconvenient'.
- Later we will discuss the use of 'thresholds' to assist with the interpretation process


## Moving onto more complex examples (the three allele locus)



There are four alleles present given a two person mixture So one person must be homozygous.
Lets work with Hp : $\mathrm{S}+\mathrm{U}$ and Hd : $\mathrm{U}+\mathrm{U}$
IF $\mathrm{S}=\mathrm{ab}$ then $\mathrm{U}=\mathrm{cc}$ under Hp then $\operatorname{Pr}(\mathrm{E} \mid \mathrm{Hp})=\mathrm{c}^{2}$
What is $\operatorname{Pr}(E \mid \mathrm{Hd})$ if the suspect is ab ?

## Denominator

| 1 | 2 | $\operatorname{Pr}$ |
| :---: | :---: | :---: |
| aa | bc | $2 \mathrm{a}^{2} \mathrm{bc}$ |
| ab | ac | $4 \mathrm{a}^{2} \mathrm{bc}$ |
| ab | bc | $4 \mathrm{ab}^{2} \mathrm{c}$ |
| bc | ab | $4 \mathrm{abc}^{2}$ |
| bb | ac | $2 \mathrm{ab}^{2} \mathrm{c}$ |
| cc | ab | $2 \mathrm{abc}^{2}$ |



## The LR (three alleles)

- Hp: S+U
- Hd:U+U
- IF $\mathrm{S}=\mathrm{ab}, \operatorname{Pr}(\mathrm{E} \mid \mathrm{Hp})=\mathrm{c}^{2}$

$$
\operatorname{Pr}(E \mid H d)=2\left(6 a^{2} b c+6 a b^{2} c+6 a b c^{2}\right)
$$

$$
L R=\frac{c^{2}}{12 a b c(a+b+c)}
$$

$$
=\frac{c}{12 a b(a+b+c)}
$$

- IF $\mathrm{S}=\mathrm{aa}, \operatorname{Pr}(\mathrm{E} \mid \mathrm{Hp})=2 \mathrm{bc}$

$$
\begin{aligned}
& \operatorname{Pr}(E \mid H d)=2\left(6 a^{2} b c+6 a b^{2} c+6 a b c^{2}\right) \\
& L R=\frac{2 b c}{12 a b c(a+b+c)} \\
& =\frac{1}{6 a(a+b+c)}
\end{aligned}
$$

## For homework try 2 alleles

- Calculations are quite time consuming
- But easily programmed
- The rules are just the same as described for any mixture and can be expanded for complex mixtures (3-person or more)

