What is a likelihood ratio?

Basic maths

- When '+' is used this mean 'OR'
- When 'x' is used this means 'AND'
- Pr. is shorthand for probability
- So the probability of a 'AND' b happening together is Pr(a and b) = $a \times b$
- And the Pr(a 'OR' b) happening = $a + b$
- Probabilities are conditional which means that the probability of something is based on a hypothesis

Conditioning

◆ In maths conditioning is denoted by a vertical bar – hence $Pr(a/b)$ means 'the probability of a **given** that b is true.

◆ This is a very important concept because the probability a is dependent upon various assumptions.

◆ The assumptions or hypotheses can change.

Conditioning

- Lets suppose that a is the probability of an event
- For example: what is the probability that it will rain in the afternoon – $Pr(a)$?
- This probability is dependent upon assumptions – look out of the window in the morning.
- $Pr(a)$ **if** it is sunny (s) is less than $Pr(a)$ **if** it is cloudy (c)
- We can write this as $Pr(a|s)$ and $Pr(a|c)$
- $Pr(s)=1-Pr(c)$

Calculation of the likelihood ratio (LR)

◆ Pr(a/s) and Pr(a/c) can be calculated from data.

◆ How often does it rain when its sunny in the morning? E.g. 20 out of 100 observations is Pr(a/s)=0.2

◆ How often does it rain when its cloudy in the morning? E.g. 80 out of 100 observations is Pr(a/c)=0.8

Formation of the likelihood ratio

◆ The LR compares these two probabilities to find out which of the two probabilities is the most likely

◆ Thus in this example we compare 2 probabilities

$$LR = \frac{\Pr(a \mid c)}{\Pr(a \mid s)} = \frac{0.8}{0.2} = 4$$

Explanation of the likelihood ratio

$$LR = \frac{\Pr(a \mid c)}{\Pr(a \mid s)} = \frac{0.8}{0.2} = 4$$

- The probability that it will rain is 4 times more likely **if** it is cloudy in the morning than **if** it is sunny in the morning
The word **if** is <u>very</u> important here. It must always be used when explaining a likelihood ratio otherwise the explanation could be misleading.

How does the LR work in crime stain work?

◆ We evaluate the evidence (E) relative to alternative pairs of hypotheses

◆ Usually, these are formulated as follows:

 ◆ Pr(E/S) the probability of the evidence **if** the stain originated from the suspect?

 ◆ Pr(E/U) the probability of the evidence **if** the stain originated from an unknown (unrelated) individual.

$$LR = \frac{\Pr(E \mid S)}{\Pr(E \mid U)}$$

The numerator

The denominator

The statement

- The probability of the evidence is x times more likely **if** the stain came from Mr Smith than **if** it came from an unknown unrelated invidual.

- It is not allowed to say: " The probability **that** the stain came from Mr Smith" because we must always include the conditioning statement – ie always make the hypothesis clear in the statement.

- Always use the 'IF' word when using a likelihood ratio to avoid this trap.